Scissors-Mode Vibrations and the Emergence of SU(3) Symmetry from the Projected Deformed Mean Field

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Starting from a deformed potential we construct separate bases of collective neutron and proton rotational states by exact angular momentum projection. These rotational states are then coupled by diagonalizing a residual pairing plus quadrupole interaction. Many new bands emerge that are not found in the rotation of the usual BCS condensate, and may correspond to the geometrical scissors mode and its generalizations. These excitation modes can be understood as rotational bands built on spin-$1\hbar$ phonon excitations; they exhibit a nearly perfect dynamical SU(3) fermion spectrum, even though there is no explicit dynamical symmetry in our model. [S0031-9007(97)05162-4]

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The long-range neutron-proton interaction is the main source of stabilization for heavy nuclear systems at finite deformation, as was realized long ago by de Shalit and Goldhaber [1], Talmi [2], and by Federman and Pittel [3]. Dobaczewski *et al.* [4] further observed that the n-p part of the quadrupole-quadrupole (Q-Q) interaction extracted from the Skyrme effective force is about 5 times larger than the n-n or the p-p parts, and that this effect is incorporated in models that assume equal neutron and proton deformations. As a consequence of this implicit strong n-p interaction, a deformed nucleus in most standard models corresponds to a rotating system with neutron and proton single-particle potentials tightly coupled in the corresponding orientation angles. Separate rotations of neutrons and protons will thus cost large amounts of energy. (An example from model results [5] exhibiting independent rotations of neutrons and protons has been asserted to be spurious [6].)

However, small perturbations of nuclear shapes and relative orientations around the equilibrium can give rise to physical states at low to moderate excitation energies. Classical examples of such motion include β and γ vibrations [7], in which neutrons and protons undergo vibrations as a collective system. These small-amplitude motions are not built into the ground state for theories like Hartree-Fock-Bogoliubov or BCS and one can only obtain the β and γ vibrations by mixing a large set of multi-quasi-particle states. A more efficient way to describe these vibrational states is to build additional correlations into the ground state, as, for example, in the random phase approximation (see, for example, Ref. [8]).

Classically, one may also consider small oscillations in the relative orientations of the neutron and proton deformed fields [9,10]. The geometric picture may be related to the two-rotor model [11]. Because of the strong restoring force [12], this oscillation is confined to a small angle between the protons and neutrons (scissors motion). This vibration, together with the β and γ vibrations, may be classified using group theoretical methods and belongs to the lowest collective excitations of the ground state, as pointed out by Iachello [13].

Early shell model work of Bhatt, Parikh, and McGrory [14] suggested that low-lying collective modes of lighter nuclei could be described as coupling of collective states of neutron and proton groups. Using the shell model to study deformed heavy nuclei microscopically is a desirable but very difficult task because of large dimensionality and its related problems. The projected shell model (PSM) provides one possible solution for this difficulty [15]. In this approach one first truncates the configuration space with guidance from the deformed mean field by selecting only the BCS vacuum plus a few quasiparticles in the Nilsson orbitals around the Fermi surface, performs angular momentum projection to obtain a set of laboratory-frame basis states, and finally diagonalizes a shell-model Hamiltonian in this space. Since the deformed mean field + BCS vacuum already incorporates strong particle-hole and particle-particle correlations, this truncation should be appropriate for the low-lying states dominated by quadrupole and pairing collectivity. Indeed, this approach has been very successful for ground band properties and near-yrast quasiparticle excitations in high-spin physics [15].

However, in this formalism the vacuum is the usual BCS condensate of neutrons and protons. Without quasiparticle excitations, one can obtain only the ground state band (g band) after angular momentum projection. There is no room for studying any other collective excitations.

In this paper we shall extend the PSM in order to study the motion in relative orientation angle between

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deformed neutron and proton fields in a microscopic way. Instead of a single BCS vacuum, the angular momentum projection is now performed for separate neutron and proton deformed BCS vacua. Although the introduction of two separately projected BCS vacua seems to treat neutrons and protons as two independent systems, the equal deformation used in the Nilsson calculation of the basis and embedded in the two BCS vacua already implies strong correlation between the two systems. The projected neutron and proton states are finally coupled through the diagonalization of a pairing plus quadrupole interaction in this basis. This procedure gives the usual ground band corresponding to a strongly coupled BCS condensate of neutrons and protons, but also leads to a new set of states arising from a more complex vacuum that incorporates fluctuations in the relative orientation of the n and p fields.

The neutron and proton valence spaces and the interaction strengths employed in the present Letter are those of Ref. [15]. Our single particle space contains three major shells ($N = 4$, 5, and 6) for neutrons and ($N = 3$, 4, and 5) for protons; this space has been shown to be sufficient for a quantitative description of rare-earth g bands and bands built on a few quasiparticle excitations [15]. The Hamiltonian [15] can be separated into $\hat{H} = \hat{H}_\nu + \hat{H}_\pi + \hat{H}_{\nu\pi}$, where H_τ ($\tau = \nu, \pi$) is the pairing plus quadrupole Hamiltonian [16], with inclusion of a quadrupole-pairing force,

$$\hat{H}_\tau = \hat{H}_\tau^0 - \frac{\chi_{\tau\tau}}{2}\sum_\mu \hat{Q}_\tau^{\dagger\mu}\hat{Q}_\tau^\mu - G_M^\tau \hat{P}_\tau^\dagger \hat{P}_\tau - G_Q^\tau \sum_\mu \hat{P}_\tau^{\dagger\mu}\hat{P}_\tau^\mu , \qquad (1)$$

$$\hat{H}_{\nu\pi} = -\chi_{\nu\pi}\sum_\mu \hat{Q}_\nu^{\dagger\mu}\hat{Q}_\pi^\mu . \qquad (2)$$

The interaction strengths $\chi_{\tau\tau}$ ($\tau = \nu$ or π) are related self-consistently to the deformation ϵ by [15]

$$\chi_{\tau\tau} = \frac{\frac{2}{3}\epsilon(\hbar\omega_\tau)^2}{\hbar\omega_\nu\langle\hat{Q}_0\rangle_\nu + \hbar\omega_\pi\langle\hat{Q}_0\rangle_\pi} . \qquad (3)$$

Obviously, neutrons and protons are coupled by the self-consistency condition. Following Ref. [15], the strength $\chi_{\nu\pi}$ of the n-p quadrupole-quadrupole residual interaction is assumed to be $\chi_{\nu\pi} = (\chi_{\nu\nu}\chi_{\pi\pi})^{1/2}$. Similar parametrizations were used in earlier works [16].

In the present initial investigation we shall not consider quasiparticle excitations. The quasiparticle vacua are defined after the BCS calculation through $|0\rangle = |0_\nu\rangle|0_\pi\rangle$. The basis for the shell model diagonalization is obtained by angular momentum projection [15] onto the vacuum:

$$|I\rangle = N^I \hat{P}^I|0\rangle \equiv N^I[\hat{P}^{I_\nu}|0_\nu\rangle \otimes \hat{P}^{I_\pi}|0_\pi\rangle]^I \equiv N^I[I_\nu \otimes I_\pi]^I, \qquad (4)$$

where $\hat{P}^I$ is the angular momentum projection operator [17] and N^I is the normalization constant. Formally, the total wave function of Eq. (4) can be expressed as $|\alpha, I\rangle = |(I_\nu; I_\pi)I\rangle$ (α distinguishes independent states having the same I).

In this way, a state with angular momentum I receives contributions from neutron and proton parts and the coupling between the two:

$$\begin{aligned}\langle\alpha, I|\hat{H}|\alpha', I\rangle &= \langle(I_\nu; I_\pi)I|\hat{H}|(I'_\nu; I'_\pi)I\rangle \\ &= [\langle I_\nu|\hat{H}_\nu|I'_\nu\rangle + \langle I_\pi|\hat{H}_\pi|I'_\pi\rangle]\delta_{I_\nu I'_\nu}\delta_{I_\pi I'_\pi} \\ &\quad - \chi_{\nu\pi}\langle(I_\nu; I_\pi)I|\hat{Q}_\nu^\dagger\hat{Q}_\pi|(I'_\nu; I'_\pi)I\rangle . \qquad (5)\end{aligned}$$

The third term in Eq. (5) can be written explicitly as

$$\begin{aligned}&\langle(I_\nu; I_\pi)I|\hat{Q}_\nu^\dagger\hat{Q}_\pi|(I'_\nu; I'_\pi)I\rangle \\ &\quad = \mathcal{W}(I_\pi 2 I I'_\nu; I'_\pi I_\nu)\langle I_\nu \| \hat{Q}_\nu \| I'_\nu\rangle \\ &\quad \times \langle I_\pi \| \hat{Q}_\pi \| I'_\pi\rangle / \sqrt{(2I_\nu + 1)(2I'_\pi + 1)} , \qquad (6)\end{aligned}$$

where $\mathcal{W}$ is the 6-j symbol.

At each spin I, we diagonalize Eq. (5). We take as a typical example the rotational nucleus, ^{168}Er, without particle number projection; results are shown in Fig. 1. The lowest band is the g band, which is nearly identical to that of earlier calculations [15] where no separation and recoupling of neutron and proton components was considered.

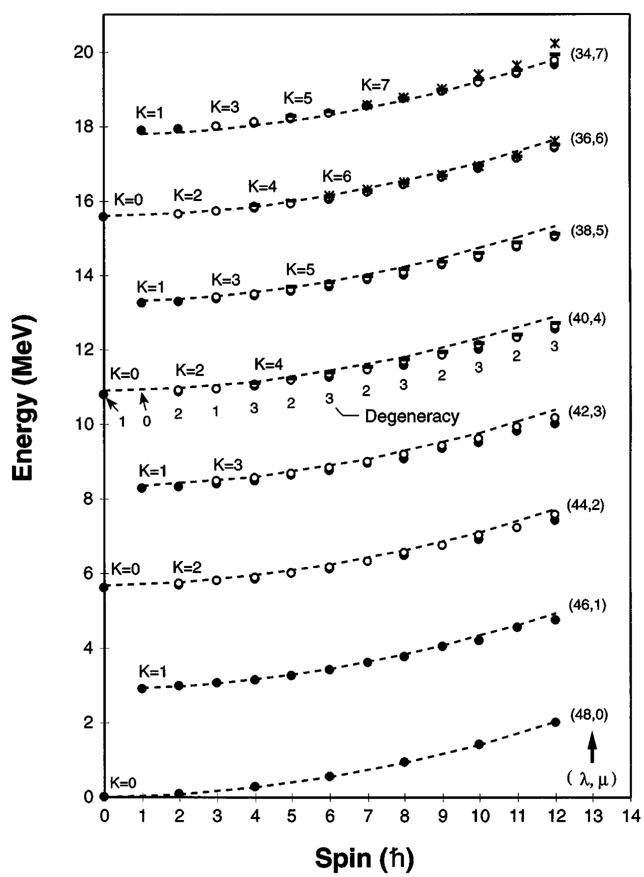

FIG. 1. Spectrum of collective excitations corresponding to coupled rotation of neutrons and protons. Symbols represent calculations using the projected shell model; lines are calculated using an SU(3) fermion dynamical symmetry model. The degeneracy is indicated explicitly for the (40, 4) states.

In addition to the g band, many new excited bands emerge that are not found in the earlier calculations. These bands exhibit a curvature similar to the g band, suggesting that they have the ground band moment of inertia.

The strikingly regular pattern of Fig. 1, which contains states up to 20 MeV in energy and $12\hbar$ in spin, can be understood as the manifestation of a nearly perfect SU(3) symmetry: all bands can be well reproduced by an SU(3) Fermion dynamical symmetry model [18] if the projected neutron and proton BCS vacuum states are considered to be two SU(3) representations coupled through the Q_n-Q_p interaction. Assuming n_ν and n_π are the effective valence neutron number and proton number, respectively, a model Hamiltonian with $\mathrm{SU}(3)^\nu \otimes \mathrm{SU}(3)^\pi \supset \mathrm{SU}(3)^{\nu+\pi}$ dynamical symmetry can be written as follows [see Eq. (3.107) in Ref. [18]]:

$$\hat{H} = \chi_n^{\mathrm{eff}} \hat{C}_{\mathrm{su3}}^{\nu} + \chi_p^{\mathrm{eff}} \hat{C}_{\mathrm{su3}}^{\pi} - \chi_{np}^{\mathrm{eff}} \hat{C}_{\mathrm{su3}}^{\nu+\pi} + \alpha \hat{J}^2, \tag{7}$$

where $\hat{C}_{\mathrm{su3}}^{\tau}$ are the $\mathrm{SU}(3)^\tau$ ($\tau = \nu, \pi$) Casimir operators for neutrons, protons, and the n-p coupled symmetry $\mathrm{SU}(3)^{\nu+\pi}$ ($\tau = \nu + \pi$).

The eigenvalue of the lowest-order SU(3) Casimir operator for a given representation (l, m) is $C(l, m) = \frac{1}{2}(l^2 + m^2 + lm + 3l + 3m)$. Assuming that the two BCS vacua correspond to the SU(3) symmetric representations $(n_\nu, 0)$ and $(n_\pi, 0)$, respectively, and that the permissible irreps of $\mathrm{SU}(3)^{\nu+\pi}$ correspond to the Littlewood rule, $(n - 2\mu, \mu)$ ($\mu = 0, 1, 2, \ldots, n_\pi$), the spectrum can be obtained analytically as

$$\begin{aligned} E - E_{\mathrm{g.s.}} &= \chi_{np}^{\mathrm{eff}}[C(n,0) - C(n - 2\mu, \mu)] + \alpha I(I+1) \\ &= \mu \hbar\omega_\infty \left[1 - \frac{\mu - 1}{n}\right] + \alpha I(I+1), \end{aligned} \tag{8}$$

where $n = n_\nu + n_\pi$ is the total effective valence particle number, $\hbar\omega_\infty = \frac{3}{2} n \chi_{np}^{\mathrm{eff}}$, and $E_{\mathrm{g.s.}}$ is the ground state energy

$$E_{\mathrm{g.s.}} = \chi_n^{\mathrm{eff}} C(n_\nu, 0) + \chi_p^{\mathrm{eff}} C(n_\pi, 0) - \chi_{np}^{\mathrm{eff}} C(n, 0). \tag{9}$$

The allowed quantum numbers are determined from the usual SU(3) subgroup reduction rules for the fermion dynamical symmetry model [18]. For example, a permissible SU(3) representation for this particle number is $(\lambda, \mu) = (40, 4)$, and this can have a $K = 0$ band with $I = 0, 2, 4, \ldots, 44$, a $K = 2$ band with $I = 2, 3, 4, \ldots, 43$, and a $K = 4$ band with $I = 4, 5, 6, \ldots, 41$ [19].

The parameters may be determined by fitting to the preceding PSM results: $n = 48$, $\hbar\omega_\infty = 2.9$ MeV, and $\alpha = 0.013$ MeV. This, coupled with Eq. (9) and the reduction rules, gives the spectrum illustrated by dashed lines in Fig. 1. States are labeled by the SU(3) irrep labels (λ, μ) and the bandhead of each rotational band within the irrep is labeled by the SU(3) quantum number K. Degeneracies at each spin may be deduced by counting one for each K band, except only even spins are present for $K = 0$ bands. We list the degeneracies of the (40, 4) representation as an example in Fig. 1. Not all PSM states can be seen clearly in the plot because of the high degeneracy, but there is a one-to-one correspondence between predicted SU(3) states and those observed in the PSM calculation. We have calculated the overlap of the PSM g-band wave functions with those obtained from an SU(3) representation $(\lambda, 0)$ as a function of angular momentum. These overlaps are found to differ from unity by only 4% or less, indicating a strong similarity of the PSM wave functions and SU(3) symmetric wave functions. Details of these calculations will be discussed in future publications.

One can see from Eq. (8) and the SU(3) reduction rules that the whole spectrum of Fig. 1 can be viewed as a set of rotational bands built on different multiphonon excitation states with the phonon energy $\hbar\omega = \hbar\omega_\infty[1 - \frac{\mu-1}{n}]$ and phonon spin $1\hbar$. For example, a three-phonon system could have two states with energy $3\hbar\omega$ and total spin $1\hbar$ and $3\hbar$; a four-phonon system could have three states with energy $4\hbar\omega$ and total spin $0\hbar$, $2\hbar$, and $4\hbar$; and so on. This provides an alternative explanation of the degeneracy of the bands obtained by the PSM diagonalization. Comparing the SU(3) and phonon classifications, we find that the SU(3) quantum numbers μ and K in Fig. 1 may be interpreted as the number of phonons and their allowed total spins, respectively.

As long as $\hbar\omega_\infty$ is held constant, the spectrum is sensitive to the effective valence particle number n only through small anharmonicities. For the present example, the phonon energy decreases smoothly from 2.9 to 2.5 MeV as the phonon number increases from 1 to 7. When $n \to \infty$, $\hbar\omega \to \hbar\omega_\infty$, and the vibration becomes harmonic.

The lowest excited band, the 1^+ band corresponding to a one-phonon excitation, warrants further discussion. In Fig. 2 we plot this band, together with the g band

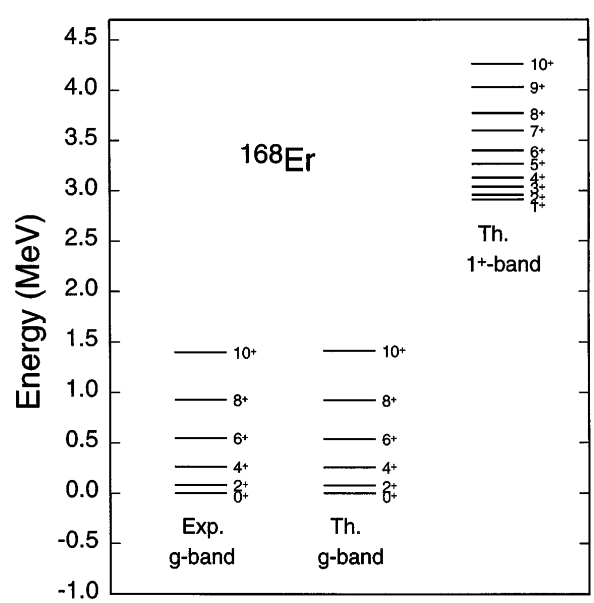

FIG. 2. The ground and 1^+ collective excited bands in ^{168}Er.

from the PSM calculations and from experiment. The excitations of the 1^+ band relative to the ground state depend on the interaction strengths used in the calculation. The good agreement of the calculated g band with data (and similar results for many other calculations in this mass region [15]) suggests that the strengths we use here are realistic. These excitations are due to an SU(3) coupling $(n_\nu, 0) \otimes (n_\pi, 0)$ in which both neutron and proton intrinsic systems remain in the ground states; thus they must be related physically to relative motion between neutrons and protons. We conclude that this may be the 1^+ scissors mode band as previously suggested in other models [9–13].

In these calculations we have not yet addressed the observed fragmentation of $M1$ strength for 1^+ states in deformed nuclei [20]. Such fragmentation could indicate that contributions from two-quasiparticle components must be taken into account [21–23], and there is controversy in the literature concerning whether these states are more economically described as collective modes or as quasiparticle states. An attempt to investigate this has been reported recently by Shimano and Ikeda [24] and by Heyde *et al.* [25].

The present results indicate that the PSM provides a microscopic framework in which collective modes that may be closely identified with those proposed in earlier geometrical and algebraic descriptions emerge as the lowest excitations. Furthermore, it is already well established that the PSM describes structures built on quasiparticle excitation very well. Therefore, the extension of the present calculations to a larger basis including two and possibly four quasiparticle excitations of the n-p coupled vacuum will provide a microscopic formalism in which collective and quasiparticle degrees of freedom enter on an equal footing. Such calculations are possible and are presently being explored. We may expect that the long-debated question of whether the observed 1^+ states are collective or two-quasiparticle in nature may then be resolved through quantitative calculation.

In conclusion, we have found many new collective modes in a shell model diagonalization based on separately projected neutron and proton Nilsson + BCS vacuum states. We have shown that these modes may be classified systematically in a phonon spectrum with weak anharmonicity. Among these states, the lowest 1^+ band at about 3 MeV may correspond to the scissors mode predicted in the classical picture. The PSM is a shell model diagonalization method that does not explicitly introduce any dynamical symmetries. However, the quantitative agreement with the SU(3) fermion dynamical symmetry model provides an algebraic fermion classification scheme for the states obtained from the PSM diagonalization, and suggests that the projected BCS vacuum for a well-deformed system has a very good effective fermion SU(3) dynamical symmetry. This in turn implies a good boson algebraic symmetry if Pauli effects may be ignored. Finally, we have proposed that the extension of the present calculations to include quasiparticle excitations can provide a quantitative framework to settle the issue of whether 1^+ states are more properly viewed as collective excitations or as quasiparticle states.

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