g factors and the interplay of collective and single-particle degrees of freedom in superdeformed mass-190 nuclei

Yang Sun,[1,2,3] Jing-ye Zhang,[1] and Mike Guidry[1]
[1]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996*
[2]*Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China*
[3]*Department of Physics, Xuzhou Normal University, Xuzhou, Jiangsu 221009, People's Republic of China*
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Interplay of collective and single-particle degrees of freedom is a common phenomenon in strongly correlated many-body systems. Despite many successful efforts in the study of superdeformed nuclei, there is still unexplored physics that can be best understood only through the nuclear magnetic properties. We point out that study of the gyromagnetic factor (g factor) may open a unique opportunity for understanding superdeformed structure. Our calculations suggest that investigation of the g-factor dependence on *spin and particle number* can provide important information on single-particle structure and its interplay with collective motion in the superdeformed mass-190 nuclei. Modern experimental techniques combined with the new generation of sensitive detectors should be capable of testing our predictions.

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The study of superdeformed nuclei has been an active focus of low-energy nuclear physics. The richness of the experimental data [1] has motivated many theoretical efforts. Most of the superdeformed yrast spectra can be well described by several models. For the mass-190 region, it is found that even though these nuclei are among the most deformed, they exhibit substantial deviation from rigid rotor behavior as the angular momentum increases, and the deviation varies from nucleus to nucleus. This deviation is a microscopic consequence of interplay between collective and single-particle motion in a nuclear many-body system. The second well-studied quantity besides the γ-ray energies, the transitional quadrupole moment, is insensitive to particular contributions from individual neutron and proton orbitals.

In a recent Letter [2], we have presented a systematic description of yrast superdeformed bands in the mass-190 region. The theoretical analysis was based on the projected shell model (PSM) [3]. Without any special adjustment in parameters, excellent agreement with available spectroscopic and transition quadrupole moment data for the series of even-even Hg and Pb isotopes was obtained. Quantitative predictions were also made for g factors in ^{194}Hg in which explicit dependence of the g factors on spin was given.

The g factor is sensitive to the single-particle structure in wave functions as well as to its interplay with collective degrees of freedom. Because of the intrinsically opposite signs of the neutron and proton g_s, a study of g factors enables determination of the microscopic structure for underlying states. For example, variation of g factors often is a clear indicator for a single-particle component that strongly influences the total wave function. One can extract g factors from experimental $B(M1)/B(E2)$ ratios, but this usually involves the introduction of several assumptions. Therefore, direct measurement of g factors is most valuable, though this is a very challenging task.

Considerable progress has been made recently in g-factor measurements [4]. By using the transient field method combined with the high-resolution detector Gammasphere, the first purported direct measurement of g factors in the superdeformed well for ^{194}Hg has been reported in a recent Rapid Communication [5]. In this paper, Mayer *et al.* concluded that the measured g factors are in agreement with the theoretical predictions of Hartree-Fock plus BCS calculations [6], as well as with the picture of a rigid rotation for which $g = Z/A$ (where Z and A are the proton and total particle number, respectively).

In the present paper, we shall point out that (1) g factors of superdeformed bands in even-even nuclei of the mass-190 region generally behave very differently from the collective Z/A estimates, and variation of g factors *as a function of spin* can provide us with information on the evolution of pair correlations; (2) *as a function of particle number*, our calculation shows another variation of g factors reflecting shell fillings, which indicates important contribution from single particles at different rotational stages. Both variations contain valuable information on superdeformed structure and certainly suggest a picture beyond rigid body rotation. However, current experimental uncertainties [5] are still too large to distinguish such features and thus do little to test theories.

Our calculation is performed by using the PSM, in which the same separable-force Hamiltonian and same configuration space are employed as in our previous Letter [2]. To treat these heavy, superdeformed nuclei, we include in our single-particle configuration space four major shells for each kind of nucleon, i.e., $N=4,5,6,7$ for neutrons and $N=3,4,5,6$ for protons. In the PSM, the many-body wave function is a superposition of (angular momentum) projected multiquasiparticle states,

$$|\psi^I_M\rangle = \sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa} |\varphi_\kappa\rangle, \tag{1}$$

where $\hat{P}^I_{MK}$ is the projection operator [7] and $|\varphi_\kappa\rangle$ denotes basis states consisting of the quasiparticle (qp) vacuum, two quasineutron and -proton, and four-qp states

$$|\varphi_\kappa\rangle=\{|0\rangle,\ a^\dagger_{\nu_1}a^\dagger_{\nu_2}|0\rangle,\ a^\dagger_{\pi_1}a^\dagger_{\pi_2}|0\rangle,\ a^\dagger_{\nu_1}a^\dagger_{\nu_2}a^\dagger_{\pi_1}a^\dagger_{\pi_2}|0\rangle\}. \tag{2}$$

In Eq. (2), the $a^\dagger$'s are the qp creation operators, ν's (π's) denote the neutron (proton) Nilsson quantum numbers that run over properly selected (low-lying) orbitals, and $|0\rangle$ the qp vacuum or zero-qp state. The dimension of the qp basis (2) is about 100 and the deformation of the basis is fixed at $\epsilon_2=0.45$ for all nuclei calculated in this paper. The choice of this deformation is based on experimental information [1]. We note that the choice of a basis deformation is not very critical for the PSM because many dynamical fluctuations are taken into account by the configuration mixing. The same Hamiltonian (including spherical single-particle, residual quadrupole-quadrupole, monopole pairing, and quadrupole pairing terms) as employed in Ref. [2] is then diagonalized in Eq. (1). The resulting energy spectrum has been compared with data as presented in [2], and the wave function $|\psi^I_M\rangle$ is used to calculate g factors:

$$g(I)=\frac{\langle\psi^I_{M=I}|\hat{\mu}_z|\psi^I_{M=I}\rangle}{\mu_N I}=\frac{\langle\psi^I||\hat{\mu}||\psi^I\rangle}{\mu_N\sqrt{I(I+1)}}, \tag{3}$$

with $\hat{\mu}$ being the magnetic vector and μ_N the nuclear magneton. $g(I)$ can be written as a sum of the proton and neutron parts $g_\pi(I)+g_\nu(I)$, with

$$g_\tau(I)=\frac{1}{\mu_N\sqrt{I(I+1)}}[g_l^\tau\langle\psi^I||\hat{j}^\tau||\psi^I\rangle+(g_s^\tau-g_l^\tau)\times\langle\psi^I||\hat{s}^\tau||\psi^I\rangle], \tag{4}$$

where $\tau=\pi$ or ν, and g_l and g_s are the orbital and spin gyromagnetic ratios, respectively. We use the free values for g_l and the free values damped by the usual 0.75 factor for g_s [8],

$$g_l^\pi=1,\quad g_s^\pi=5.586\times0.75,$$
$$g_l^\nu=0,\quad g_s^\nu=-3.826\times0.75. \tag{5}$$

We emphasize that g factor in the PSM is computed directly from the many-body wave function [Eqs. (1)–(4)] without a semiclassical separation of the collective and the single-particle parts. In particular, there is no need to introduce a core contribution, g_R, which is a model-dependent concept and not a measurable quantity. Previous g-factor calculations of the PSM have been tested by experiment. For example, predictions for the normally deformed rare-earth nuclei [3,9] were supported by later measurement [10].

Our calculated g factors for ^{194}Hg (filled diamonds) are shown in Fig. 1. In the low spin range, we observe a smooth increase in the g factors up to the spin states around $I=26$. Bengtsson and Åberg suggested that an increase in g factor at low spins was due to changes in deformation and pairing [11]. Since deformation does not have a large variation along a band in this heavy, well-superdeformed nucleus ($<2\%$ as seen from our calculation in [2] and from relativistic [12] and nonrelativistic [13] mean field theories), the g-factor increase in the low spin range can be attributed to the pairing change caused by the Coriolis antipairing effect, which coherently weakens pair correlation across many pairs. The weakened pairing was demonstrated in our calculations for pairing gaps in [2] and was suggested as the main source for the increase in the moment of inertia before $I=26$.

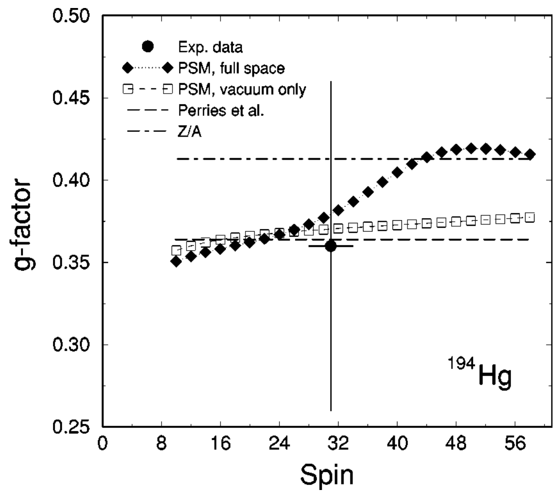

FIG. 1. g factors for the superdeformed yrast band of ^{194}Hg. Filled diamonds are the PSM results with the full model space, open squares the PSM results calculated in the projected zero-qp state only, filled circle (with error bars) the experimental data [5], dashed line the HF+BCS result [6], and dashed-dotted line the rigid-body value.

Above the spin $I=26$, an accelerated increase is seen for the range $I=26$–44. At $I=44$, the g factor reaches a value of $0.41\approx Z/A$. This behavior suggests that rotation alignment of high-j orbitals plays an additional role in this spin range, adding contributions to the g-factor increase in addition to that from the coherently weakened pairing. This is consistent with the mechanism that we suggested [2] to explain the observed gradual increase in the dynamical moment of inertia in superdeformed mass-190 nuclei. The rotation alignment of high-j orbitals enhances the moment of inertia, but the high-j alignment contribution alone seems insufficient to cause the pronounced increase in the moment of inertia seen in ^{194}Hg. In particular, this cannot easily explain the increase in the moment of inertia before $I=26$, where the high-j pair alignment is negligibly small. This combined effect of high-j alignment and decrease of pairing is also seen in cranked relativistic Hartree-Bogoliubov theory [12].

The g factor measured by Mayer *et al.* [5] is plotted in Fig. 1 for comparison. Because the error bar is large, it is impossible to draw a definite conclusion. However, it is interesting to note that the experimentally suggested spin range for this measurement is such that the rotation alignment of high-j orbitals is just beginning to contribute. Therefore, the current measurement lies in a sensitive spin range where one could have a chance to track the competition of the Coriolis antipairing and the high-j pair alignment. A test of these ideas will require reduction of the experimental uncertainty by a factor of at least 2–3 and extension of the data to at least spin states $I=36$-42.

Collective g factors were calculated systematically by

Perries *et al.* [6] using the Hartree-Fock plus BCS theory with the SkM* effective force. Since the dependence of g factors on spin was not given in their calculation, the results may be considered as a band average over the low spin states. The g factor obtained for ^{194}Hg by Perries *et al.* [6] is also shown in Fig. 1 for comparison (dashed line). It is obviously very different from the Z/A value (dashed-dotted line), which was the main conclusion and was clearly stated in [6]. The value of Perries *et al.* matches well with our calculations for the spin range from the measured band head up to the $I=26$ state. However, the two calculations diverge after that spin as the PSM results receive contributions from the high-j pair alignment.

To see this clearly, we have made a test calculation with the PSM in which all the multi-qp configurations in Eq. (2) are excluded except the vacuum (zero-qp state). This effectively removes contributions of any pair alignment to the total wave function. The results are shown in Fig. 1 as open squares. As can be seen, they are close to that of Perries *et al.*, although a visible increase along the band still exists in the PSM calculation, which is the result caused by the Coriolis antipairing effect as discussed before. However, these results are not from our complete shell model basis and the important ingredients in the basis, which can correctly describe the higher spin behavior of the moments of inertia including the observed downturning [14] at the highest spins, are now excluded. It is seen, from the PSM results in Fig. 1, the rigid-body g-factor value Z/A in this case can only be reached at the highest spin states when the full space in Eq. (2) is employed.

Thus, although these heavy nuclei have large deformations and the deformation is very stable against rotation, they are not simple rigid rotors. Because this cannot be seen obviously from the spectroscopy and transition quadrupole moments, accurate measurement of g factors may play a unique role in exploring the microscopic structure, as we shall further demonstrate below.

It is well known that high-j particles that lie in orbits close to Fermi levels and most easily align their spins along the axis of system rotation may have significant influence on g factors if the pair correlation among them is broken. In the superdeformed well of the mass-190 Hg and Pb nuclei, $K=\frac{3}{2}$ and $\frac{5}{2}$ protons in the $i_{13/2}$ orbital, and $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ neutrons in the $j_{15/2}$ orbital are such particles. We expect that changing the single-particle environment around the Fermi levels for different shell filings will lead to a different pattern of g factors. Figure 2 shows the g factors of superdeformed yrast bands calculated by the PSM for the Hg and Pb isotopes with neutron numbers from 108 to 116. The wave functions used to calculate these g factors are obtained when solving the eigenvalue equations, with the corresponding energies having been presented in Figs. 1 and 2 of Ref. [2].

Three observations can be made immediately from Fig. 2: (1) For all nuclei presented, the g factors clearly deviate from the corresponding rigid-body value Z/A. For the Hg isotopes, there are narrow spin ranges where the g factors can be close to Z/A (but such spin ranges are very distinct in different isotopes). However, for the Pb isotopes, the g factors are systematically smaller than the corresponding rigid-body value for all spin states calculated. We note that a smaller g factor for Pb isotopes relative to Hg has also been seen in the effectively spin-averaged calculations of Perries *et al.* [6]. (2) For the spin range where g factors in ^{194}Hg were measured, a clear increase in g factors as a function of spin is predicted for the Hg isotopes, but in Pb, they are nearly constant. The rate of increase in the Hg isotopes is obviously different, with the smallest seen in ^{192}Hg. (3) A drop in g factors is predicted at high spins ($I=44$–48) for the lightest isotopes.

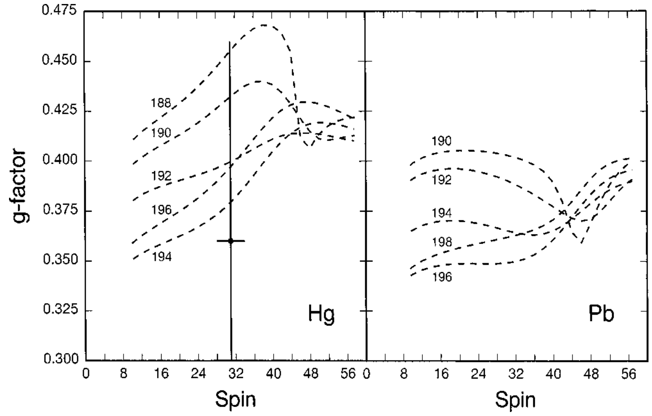

FIG. 2. Predicted g factors for superdeformed yrast band of $^{188-196}$Hg and $^{190-198}$Pb.

These results can be understood as consequences of competition between the high-j neutron and proton particles. Note that the yrast band is obtained by band mixing from the unperturbed bands with the lowest one in energy having the largest contribution. By analyzing the wave functions, we find that the two-quasiproton band ($\pi i_{13/2}[\frac{3}{2},\frac{5}{2}]$, $K=1$) is lower in the Hg isotopes, and thus closer to the zero-qp band. This increase in the importance of proton components makes the g factors in the Hg isotopes systematically larger than those in the Pb isotopes. As spin increases, this two-quasiproton band approaches the zero-qp band, leading the g-factor increase. However, depending on the positions of two-quasineutron bands, influence of the two-quasiproton band can be largely compensated if sufficient single-neutron components are mixed in the wave function. The relevant two-quasineutron bands have the structure $\nu j_{15/2}[\frac{1}{2},\frac{3}{2}]$ and $\nu j_{15/2}[\frac{3}{2},\frac{5}{2}]$, both coupled to $K=1$. For lighter isotopes with neutron numbers $N=108$ and 110, these two two-quasineutron bands approach the zero-qp band at higher spins and compete with the two-quasiproton band, thus can have large influence in the wave function for the higher spin states. This leads to a drop in g factors at $I=44$–48. For the heavier isotopes, however, only one two-quasineutron band ($\nu j_{15/2}[\frac{3}{2},\frac{5}{2}]$) is low enough in energy to compete with the two-quasiproton band. Therefore, the g-factor drop at high spins in the heavier isotopes is much diminished.

In Table I, we take ^{190}Hg as an example and show the coefficients f_κ in the yrast wave function [see Eq. (1)] for four low-lying configurations that strongly govern the evolution of g factors. While the zero-qp band dominates absolutely the wave function for low spin states, the two-qp com-

TABLE I. Wave functions f_κ [see Eq. (1)] for the superdeformed yrast band of ^{190}Hg. Only components of the zero-qp, two-qp proton band ($\pi i_{13/2}[\frac{3}{2},\frac{5}{2}]$, $K=1$), two-qp neutron band I ($\nu j_{15/2}[\frac{1}{2},\frac{3}{2}]$, $K=1$), and two-qp neutron band II ($\nu j_{15/2}[\frac{3}{2},\frac{5}{2}]$, $K=1$) are shown. Note that, because of nonorthogonality of the projected basis, only the relative size of the numbers for a given spin is significant.

Spin I	Zero-qp band	Two-qp π band	Two-qp ν band I	Two-qp ν band II
10	0.4235	0.0285	0.0315	0.0119
14	0.3692	0.0366	0.0349	0.0155
18	0.3666	0.0505	0.0396	0.0217
22	0.3980	0.0743	0.0454	0.0326
26	0.4589	0.1154	0.0521	0.0522
30	0.5452	0.1857	0.0595	0.0875
34	0.6472	0.3017	0.0697	0.1503
38	0.7499	0.4827	0.0936	0.2613
42	0.8343	0.7510	0.1649	0.4600
46	0.8450	1.1046	0.3718	0.8117
50	0.6607	1.4235	0.8324	1.3317
54	0.2782	1.7152	1.5114	2.0016

ponents become more important in the wave functions as spin increases. Comparing the relative size of the numbers at each spin, we see a much faster increase of the two-quasiproton band. This suggests a dominant role for the protons over the neutrons in the states with low and intermediate spins. For higher spin states, the combined contribution of the two two-quasineutron bands becomes important and eventually this dominates the wave function for states at the highest spins.

We conclude that experimental information on the g factor would be very valuable for exploring the underlying microscopic inside of the superdeformed nuclei. Unfortunately, the currently measured g factor [5] is not accurate enough to make a meaningful comparison with theory. As we can see in Fig. 2, the experimental error bar extends over the theoretical predictions in all the five Hg isotopes, and thus can make no distinction for the underlying microscopic structure discussed above. To begin to make meaningful comparisons, the g-factor measurement uncertainties must be reduced by at least a factor of 2–3, and measurements extended to higher spin states.

In summary, our understanding of the microscopic structure for the superdeformed mass-190 nuclei is not complete and the quantities that can distinguish physics as functions of spin and particle number have not been well studied. The g factor is such a quantity that contains information on the interplay of collective and single-particle degrees of freedom. The problem discussed here specifically for mass-190 nuclei may be common for other superdeformed mass regions. New experimental techniques such as those discussed in Ref. [4] hold the promise of permitting these prospects to be tested thoroughly for the first time.

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[1] X.-L. Han and C.-L. Wu, At. Data Nucl. Data Tables **73**, 43 (1999).
[2] Y. Sun, J.-y. Zhang, and M. Guidry, Phys. Rev. Lett. **78**, 2321 (1997).
[3] K. Hara and Y. Sun, Int. J. Mod. Phys. E **4**, 637 (1995).
[4] R. Ernst *et al.*, Phys. Rev. Lett. **84**, 416 (2000).
[5] R.H. Mayer *et al.*, Phys. Rev. C **58**, R2640 (1998).
[6] S. Perries *et al.*, Phys. Rev. C **55**, 1797 (1997).
[7] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
[8] A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975).
[9] Y. Sun and J.L. Egido, Nucl. Phys. **A580**, 1 (1994).
[10] F. Brandolini *et al.*, Nucl. Phys. **A600**, 272 (1996); Eur. Phys. J. A **6**, 149 (1999).
[11] R. Bengtsson and S. Åberg, Phys. Lett. B **172**, 277 (1986).
[12] A.V. Afanasjev, J. König, and P. Ring, Phys. Rev. C **60**, 051303(R) (1999).
[13] A. Valor, J.L. Egido, and L.M. Robledo, Nucl. Phys. **A665**, 46 (2000).
[14] B. Cederwell *et al.*, Phys. Rev. Lett. **72**, 3150 (1994).